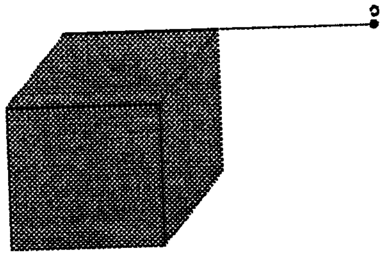
Canada et USA : Extraits des épreuves de sélection et des sujets de préparation à l'université de Toronto (Canada).

Des données numériques utiles sont placées en bas de la page 3. On rappelle qu'en anglais, le séparateur décimal est un point et que la virgule est un séparateur des milliers.

Canada 2000 Préparation Problème 2

Cubic quandry



a) How much time does it take to increase the temperature of a perfectly black cube, of dimension L on each side, by 1°C, using a lamp of power Q (J s^{-1}) located at distance L from one of the corners, along the direction of an edge (at point O on the figure). Assume that the lamp is emitting uniformly and the cube absorbs all wavelengths of light perfectly, but loses no energy by radiation or conduction elsewhere.
b) Really, what happens, is that the black cube is also re-emitting energy as a *blackbody radiator.* Now, adjust your answer and tell exactly how much time will it take.
c) Can you calculate the final temperature of the cube after a long time?
You should assume that the cube is a perfect conductor.

Canada 2003 Préparation Question 3

Temperature of the planet Venus
All objects emit radiation. The energy flux density J of this radiation is the energy emitted per unit time and per unit area. The Stefan-Boltzmann law is $J=e\sigma T4$, where $e=1$ for a perfect radiator, σ is a constant, and T is the temperature of the object. What is the equilibrium temperature of the planet Venus? Assume that the only radiation incident on Venus is from the Sun. Assume that Venus and the Sun are perfect radiators. If your calculated T is different from the measured temperature of Venus of about 700 Kelvin, speculate on why this is the case.

Mean Sun-Venus distance	R_{SV}	$1.08\ 10^{8}$ km
Temperature of the Sun	T_S	5800 K
Radius of the Sun	R_S	$6.96\ 10^{5}$ km

Canada 1999 Préparation Problème 1

Pushy photons
In an episode of *Star Trek: Deep Space 9*, Capt. Benjamin Sisco makes a hobby out of recreating an early spacecraft, from alien archaeological records. This spacecraft uses huge sails to move the ship along on the solar wind (a stream of charged particles) or by light pressure from the sun. Consider such a craft the same distance from the sun as is the earth, and only consider light pressure.
a) What is the best colour for the sail?
b) For a sail area the size of a soccer field on a small ship of 2,000 kg, how fast will the ship be moving after 24 hours? After 1 year? In my research laboratory we have a laser which produces a terawatt of power (10^{12} W) in a pulse of light about one picosecond (10^{-12} s) duration. This pulse can be focussed to a spot about 10 µm across.
c) How much force from light pressure does such a pulse exert? Roughly how much momentum is transferred? What is the pressure exerted by the light?
N.B. Au niveau de l'orbite terrestre, la lumière solaire a une puissance surfacique $P_S = 1353$ Wm^{-2}.

Canada 2005 Sélection Question 6

Si la distance moyenne Terre-Soleil était augmentée de 1% (orbite quasi-circulaire)
a) Quelle serait la nouvelle durée de l'année ?
b) Quelle serait la nouvelle température moyenne de la surface terrestre ? (Elle vaut actuellement 287 K)
Conseil : l'utilisation de données numériques inutiles relatives au système solaire est *fortement* déconseillée.

Canada 2005 Préparation problème 3 Solution

In general, the momentum vector diagram for the photon scattering from motionless electron (Compton effect) is shown in the fig. 3.1: $p'_{electron}$ is momentum of the electron after the scattering; p_{photon} is momentum of the photon before the scattering; p'_{photon} is momentum of the scattered photon.

In addition to the law of conservation of momentum, we can use the law of conservation of energy taking into account that energy of photon is proportional to the photon's frequency ν and inversely proportional to the wavelength λ_{photon}:

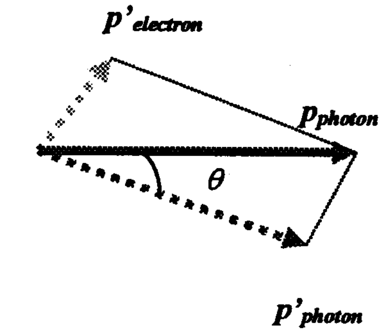


Fig.3.1

$$E_{photon} = h\nu = \frac{hc}{\lambda_{photon}},$$

where h is the Plank's constant.

The law of conservation of energy for the system of electron and photon gives the following:

$$K = h\nu - h\nu' = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) \qquad (3.1)$$

We can combine it with the given equation for the Compton wavelength shift:

$$\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta) \qquad (3.2)$$

The angle of scattering θ is not given, but we can determine it from the condition of the maximum kinetic energy of the electron after scattering. The maximum electron kinetic energy corresponds to the maximum difference of the frequencies in eq.3.1. For a photon $h\nu = pc$. Thus, the maximum kinetic energy of electron corresponds to the maximum difference of the momentum of the initial and scattered photon. Fig.3.1 helps to understand that this occurs when $\theta = 180^\circ$, and $\cos\theta = -1$. Thus, eq.3.2 may be replaced by the new one:

$$\lambda' - \lambda = \frac{2h}{mc} \qquad (3.3)$$

Let us find λ' from eq.3.3 and substitute it in eq.3.1 for maximum kinetic energy of electron:

$$\frac{K_{max}}{hc} = \frac{1}{\lambda} - \frac{1}{\lambda + \frac{2h}{mc}} = \frac{2h}{mc\lambda\left(\lambda + \frac{2h}{mc}\right)}$$

The quadratic equation for the unknown wavelength is:

$$\lambda^2 + 2\frac{h}{mc}\lambda - \frac{2h^2}{mK_{max}} = 0 \qquad (3.4)$$

From the two possible solutions for the eq.3.4 we should choose only the positive one:

$$\lambda = \frac{h}{mc}\left(\sqrt{1 + \frac{2mc^2}{K_{max}}} - 1\right) = \lambda_C\left(\sqrt{1 + \frac{2mc^2}{K_{max}}} - 1\right) \qquad (3.5)$$

where $\lambda_C = 0.00243\text{nm} = 2.43\text{x}10^{-12}\text{m}$ is the Compton wavelength of electron.

To simplify calculations, we can find the value of the rest energy of electron under the root of eq.3.5 in electronvolts: $mc^2 = 0.511$ MeV (is given in all physics handbooks and textbooks).

$$\lambda = 2.43\times10^{-12}m\left(\sqrt{1 + \frac{2\times0.511}{0.19}} - 1\right) = 3.71\times10^{-12}m = 3.71pm$$

Canada 2003 Sélection Question 5

(Background for Question 5) All objects emit electromagnetic radiation known as thermal radiation. For example, the filament of a light bulb (heated by electric power) emits electromagnetic radiation, part of which is light. The total power, P, radiated in light and other electromagnetic waves can often be described by Stefan-Boltzmann law:

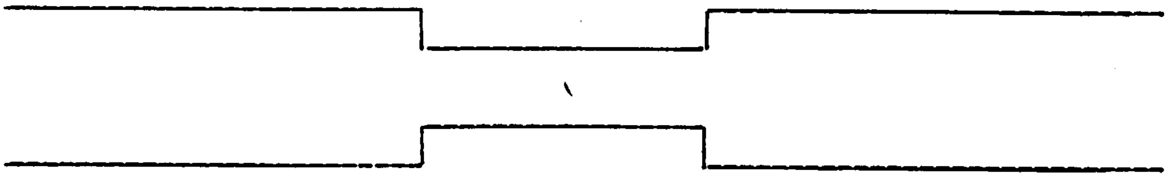
$$P = \sigma A T^4,$$

where T is absolute temperature of the body in Kelvin, A is its surface area, and σ is some constant.

The hot filament of a bulb (a piece of tungsten wire) is in energy balance, so that at each second the amount of heat energy resulting from electric current flowing through the wire is exactly equal to the amount of energy emitted from the filament via thermal radiation.

When a light bulb is on, tungsten evaporates slowly from the filament. Since the filament is not perfectly uniform, tungsten may evaporate more quickly from some pieces of the filament than from others, leaving some parts thinner. This results in a temperature increase in the thin parts, making evaporation occur even more quickly, since the rate of evaporation is higher when the temperature is higher. Eventually, as a piece of the filament becomes progressively thinner and hotter and thinner (and so on), the temperature is high enough to melt that part and the bulb breaks.

(5) Consider the following simple model. Assume that the filament is a straight uniform cylindrical wire with constant diameter except that a piece of it has half the diameter of the rest of the wire (as shown in the figure below). Assume that the length of each piece is much larger than its diameter so that one can neglect the "boundary" effects at the points where the wire changes diameter. Estimate the temperature of the thin part of the filament if the temperature of the thick part is 2000 K. Assume that the temperature is constant within each part and changes suddenly between parts. Assume that the resistivity of both parts is the same.



USA 2003 Question 30

One object has twice the temperature of a second identical object. How does the rate R_1 at which the first object radiates energy compare to the rate R_2 at which the second object radiates energy?

A) $R_1 = R_2$ B) $R_1 = 2R_2$ C) $R_1 = 4R_2$ D) $R_1 = 8R_2$ E) $R_1 = 16R_2$

USA 2003 Question 29

Assume that sodium produces monochromatic light with a wavelength of 5.89×10^{-7} m. At what rate would a 10 watt sodium vapor light be emitting photons?

A) 3.5×10^{19} photons/s
B) 3.0×10^{19} photons/s
C) 2.5×10^{19} photons/s
D) 2.0×10^{19} photons/s
E) 1.5×10^{19} photons/s

USA 1997 Problème A4

An atom in an excited state typically decays by emitting a photon. The atom is originally in some excited state S', and after the decay it is in the ground state S. We usually think of the photon as carrying away the energy difference between states S' and S. In this problem we examine this process closely.

(a, 4) Consider a stationary, isolated atom in some excited state S'. Before decay the mass of the atom is M'. The atom decays into the ground state S. In the ground state the mass of the atom is M. Thus the energy $(M'-M)c^2$, which we denote here as E_o, is the energy that is released by the atom, and is therefore an upper limit on the energy that the photon could possibly carry away (c is the speed of light). Let E denote the energy of the outgoing photon. Without solving any equation, by merely giving an qualitative argument, explain why E must be strictly less than E_o in the case of an isolated excited atom undergoing radiative decay.

(b, 6) Quantify your answer to part (a) by showing that, approximately,

$$E \approx E_o[\,1 - (E_o / 2Mc^2)\,], \text{ where } E_o / 2Mc^2 \ll 1.$$

(c, 5) Consider an atom decaying from the excited state S' to the ground state S, where $Mc^2 = 2\times10^{11}$ eV and $E_o = 4\times10^5$ eV. A photon energy corresponds to a radiation frequency. Thus the energy difference $E_o - E$ corresponds to a frequency shift $\Delta f = f_o - f$. Calculate the fractional frequency shift $\Delta f / f_o$.

(d, 5) Another way to produce a frequency shift is the Doppler effect, produced by relative motion between source and observer. The atom of part (c) emits the photon energy E, which corresponds to light frequency f. With what speed and in what direction would an observer have to move relative to this photon, to Doppler-shift its frequency back to the value f_o corresponding to the energy E_o? The observer's speed v is $\ll c$.

Possibly Useful Information

Gravitational field at the Earth's surface	$g = 9.8$ N/kg
Newton's gravitational constant	$G = 6.67 \times 10^{-11}$ N·m²/kg²
Coulomb's constant	$k = 1/4\pi\varepsilon_o = 9.0 \times 10^9$ N·m²/C²
Biot-Savart constant	$k_m = \mu_o/4\pi = 10^{-7}$ T·m/A
Speed of light in a vacuum	$c = 3.0 \times 10^8$ m/s
Boltzmann's constant	$k_B = 1.38 \times 10^{-23}$ J/K
Avogadro's number	$N_A = 6.02 \times 10^{23}$ (mol)$^{-1}$
Ideal gas constant	$R = N_A k_B = 8.31$ J/(mol·K)
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}$ J/(s·m²·K⁴)
Elementary charge	$e = 1.6 \times 10^{-19}$ C
1 electron volt	1 eV $= 1.6 \times 10^{-19}$ J
Planck's constant	$h = 6.63 \times 10^{-34}$ J·s $= 4.14 \times 10^{-15}$ eV·s
Electron mass	$m = 9.1 \times 10^{-31}$ kg $= 0.51$ MeV/c²

Canada 2005 Préparation

Estimate the temperature on the surface of the Sun, assuming that the average surface temperature on the Earth is around 20 degrees (Centigrade), radius of the Sun is about 1 percent of the distance from the Sun to the Earth, and the radiation power from the unit of surface is proportional to the fourth power of the surface temperature (Stefan-Boltzmann law).

Canada 2005 Préparation problème 4

In 1923, in his doctoral dissertation, Louis Victor de Broglie postulated that, "*because photons have wave and particle characteristics, perhaps all forms of matter have wave as well as particle properties*". De Broglie suggested that material particles of momentum p should also have wave properties and a wavelength of

$$\lambda = \frac{h}{p} \text{ (the de Broglie wavelength),}$$

where h is Planck's constant. The frequency of a particle is $f = E / h$. The further numerical experiments with electron diffraction proved the de Broglie hypothesis.

A beam of electrons accelerated by the potential difference $V = 25$ V is normally incident on a two-slit barrier with the slit separation of $d = 50\ \mu$ m.

Find the distance between two adjacent maximums of the diffraction pattern on the screen which is separated from the barrier by $l = 100$ cm.

Canada 2005 Préparation problème 3

In 1923 Arthur Holly Compton in Chicago discovered one of the most famous phenomena that proved the particle-wave dualism for all natural objects. It was found that the scattering of photons from electrons could be explained by treating photons as point-like particles with energy hf ($h = 6.63\text{x}10^{-34}$J is Planck's constant; f is a photon's frequency) and momentum hf/c (c is speed of light in free space). Such scattering may be considered a collision, in which the energy and momentum of the isolated system of the photon and the electron are conserved. The Compton shift equation gives the change in the photon's wavelength after scattering on any particle, and the new direction of photon after the scattering:

$$\lambda' - \lambda_0 = \frac{h}{mc}(1 - \cos\theta),$$

where λ' is the changed wavelength of a photon; λ_0 is the initial photon's wavelength; m is the mass of the particle; θ is the angle between the changed and the initial direction of the motion of a photon (angle of scattering). The coefficient $\frac{h}{mc}$ is called the Compton wavelength of the particle. For electron it is equal to 0.00243 nm.

Find the wavelength of the x-rays before scattering on the motionless electrons if the maximum kinetic energy of the electrons after the Compton scattering is 0.19 MeV.

Canada et USA : Solutions

Canada 2000 Préparation Problème 2

Cubic quandry

a) There are 6×4 = 24 equal cubes that can perfectly surround the lamp.so we can say that the fraction of power heating one cube is 1/24.

$$\frac{1}{24}Qt = mc\Delta\theta$$

$$?\quad t = \frac{24mc}{Q}\ \text{[seconds]}$$

b) A unit area of a blackbody radiator in temperature T, emits energy by the power of σT^4 Watts. so our cube is emitting $6L^2\,\sigma\,T^4$ J/s by the assumption that our cube is in thermal equilibrium each time. Consider our case that any small change in T, will affect the emission rate and the time needed to increase the temperature accordingly. But, in this case, we are looking at changing the temperature by 1K at room temperature, i.e., from 300K to 301K. The relative change is on the order of 1/300. and the change in T^4 is about $4 \times (1/300)$.

$$301^4 = (300+1)^4 = 300^4\left(1+\frac{1}{300}\right)^4 \cong 300^4\left(1+4\cdot\frac{1}{300}\right)$$

To a good approximation, we can ignore the changes in dT/dt due to a small change in T, and we can substitute the room temperature instead.

$$-6L^2\sigma T^4 t + \frac{1}{24}Q t = mc\,\Delta\theta$$

$$\Rightarrow t = \{mc\}/\left\{\frac{Q}{24} - 6L^2\sigma T^4\right\}$$

c) In thermal equilibrium, the emission rate and heating rate are equal. so we have :

$$-6L^2\sigma T^4 = \frac{1}{24}Q$$

$$\Rightarrow T = \left\{\frac{Q}{144\sigma L^2}\right\}^{1/4}$$

[Amir]

Canada 2003 Préparation Question 3

The energy flux density emitted by the Sun is $J_S = \sigma T_S^4$. This gives the energy per unit time flowing through unit area. The total energy per unit time going through the surface of the Sun is the energy flux density times the surface area of the Sun, namely, $(4\pi R_S^2)J_S$. If there is no energy lost as this radiation propagates between the Sun and Venus, then the same amount of energy per unit time must reach the orbit of Venus. In equilibrium, the radiation that Venus intercepts equals the amount it emits.

$$(4\pi R_S^2)\sigma T_S^4 \frac{\pi R_V^2}{4\pi R_{SV}^2} = (4\pi R_V^2)\sigma T_V^4 \,, \tag{9}$$

$$T_V^4 = \frac{R_S^2}{4R_{SV}^2} T_S^4 \,, \tag{10}$$

$$T_V = \sqrt{\frac{R_S}{2R_{SV}}} T_S \,. \tag{11}$$

Substituting for R_S, R_{SV}, and T_S gives $T_V \approx 330$ Kelvin. This is lower than the measured temperature of Venus of about 700 Kelvin. The atmosphere of Venus contains a large amount of carbon dioxide. This is a “greenhouse” gas. It traps the radiation re-emitted by the planet and leads to a higher surface temperature. Could this effect happen on Earth?

Canada 1999 Préparation Problème 1

Pushy Photons

a) Black is good — it would absorb all the light, and therefore transfer all the momentum of the light. But *silver* would be better — when the light reflects back to where it came from, it reverses its momentum. So this would mean (p– (–p)) = 2p for the momentum transferred to the sail.

b) From the useful bits of info at the bottom of the question sheets, the solar constant is 1.353 kW m^{-2}. For light, E = pc, so by taking derivatives we also have that:

$$\frac{dE}{dt} = \frac{dp}{dt} \cdot c$$

However, *power* P is defined as the rate of change of energy with time, on the left-hand side, and *force* F is defined as the rate of change of momentum with time, on the right-hand side. So we can write:

$$P = \frac{dE}{dt} = \frac{dp}{dt} \cdot c = F \cdot c$$

or

$$F = \frac{P}{c}$$

or actually F = 2P/ c if we let the light be reflected.

A soccer field isn't always the same size, but has to be in a certain allowed range of sizes; roughly 100m × 50m might be reasonable. The power P incident on the field is intensity times area: $P = IA = 1.353 \text{ kW m}^{-2} \bullet 5000 \text{ m}^2 = 6.765 \text{ MW}$ (quite a bit of power), so the force on the whole large area would be

$$F = 2 \bullet 6.765 \text{ MW} \;/\; (3 \times 10^8 \text{ m s}^{-1}) = 4.51 \times 10^{-2} \text{ N}$$

Also, F = ma, so the acceleration of the 2,000 kg spacecraft would be:

$$a = 2.25 \times 10^{-5} \text{ m s}^{-2}$$

The velocity after constant acceleration for a time t depends on initial velocity (which is zero, here):

$$v = u + at$$

A day of seconds is (24 hr) • (60 min/hr) • (60 sec/min) = 86,400 s. So the velocity after a day would be 1.94 m s^{-1}, which is jogging speed. After a year, it would be 365 times this value, about 710 m s^{-1}, which is roughly 10% of the orbital speed of a low-orbit satellite.

c) Again, if the light is reflected, $F = 2P/c = 2 \bullet 10^{12} \text{ W} / (3 \times 10^8 \text{ m s}^{-1}) = 7 \times 10^3$ N, which is the gravitational force of the smallest automobile. The force is only applied for a trillionth of a second, so the momentum transfer (the impulse) is:

$$p \bullet \Delta t = (7 \times 10^3 \text{ N}) \bullet (10^{-12} \text{ s}) = 7 \times 10^{-9} \text{ kg m s}^{-1}$$

which is small, like a big dust-mote moving in a sunbeam, or a raindrop in a very misty rain.

The *pressure* is a different story, though, because this force is applied over a tiny spot:

$$P = F / A = (7 \times 10^3 \text{ N}) / ((10 \times 10^{-6} \text{ m})^2) = 7 \times 10^{13} \text{ N m}^{-2}$$

$100 \text{ kPa} = 100 \times 10^3 \text{ N m}^{-2}$ is roughly one atmosphere of pressure. So this pressure, briefly exerted by the focussed laser pulse, is something like 700 million atmospheres of pressure. *[Robin]*

Canada 2004 Sélection Question 6

a) 3e loi de Kepler $\dfrac{T^2}{a^3} = \text{cte} \Rightarrow \dfrac{\Delta T}{T} = \dfrac{3\Delta a}{a}$

b) Voir la solution de la question 3 de préparation 2003 formule (11) $T_v \propto \dfrac{1}{\sqrt{R_{SV}}}$

donc $\dfrac{\Delta \text{ Température}}{\text{Température}} = -\dfrac{1}{2}\dfrac{\Delta a}{a}$

Canada 2003 Sélection Question 5

Let the subscript 1 refer to the thick part of the wire and 2 to the thin part of the wire. Then, the diameter d, surface area A, and cross-sectional area a of two pieces of wire of length ℓ are related by $d_1 = 2d_2$, $A_1 = 2A_2$, and $a_1 = 4a_2$. Since the resistance of a piece of wire is proportional to its resistivity and length and inversely proportional to its cross-sectional area, the resistance per unit length of the thin wire is 4 times as great as that of the thick wire. The power dissipated by a current through a resistor is $P = I^2R$, so the thin wire dissipates power at 4 times the rate of the thick wire. From the Stefan-Boltzmann law,

$$P_2 = \sigma A_2 T_2^4 \,,$$
$$(4P_1) = \sigma(A_1/2)T_2^4 \,,$$
$$P_1/(A_1\sigma) = T_2^4/8 \,,$$
$$T_1^4 = T_2^4/8 \,,$$
$$T_2 = 8^{1/4}T_1 = (1.68)(2000 \text{ K}) = 3364 \text{ K} \,.$$

USA 2003 Question 30 E

USA 2003 Question 29 B

USA 1997 Problème A4

a) Le photon a une quantité de mouvement $p = \frac{E}{c}$
Pour conserver la quantité de mouvement du système global, l'atome doit reculer (quantité de mouvement opposée à celle du photon. $E_0 = E + E_{recul}$

$E_{recul} = \frac{p^2_{atome}}{2M} > 0$ donc $E < E_0$

b) $E_{recul} = \frac{p^2}{2M} = \frac{E^2}{2Mc^2} \Rightarrow E_0 = E + \frac{E^2}{2Mc^2} = E\left(1 + \frac{E}{2Mc^2}\right)$

$E = \frac{E_0}{1+\frac{E}{2Mc^2}} \sim E_0\left(1 - \frac{E}{2Mc^2}\right) \sim E_0\left(1 - \frac{E_0}{2Mc^2}\right)$

c) $E = hf \Rightarrow \frac{\Delta E}{E} = \frac{\Delta f}{f} = \frac{E_0}{2Mc^2} = 10^{-6}$

d) Effet Doppler $\frac{\Delta f}{f} = \frac{v}{c} \Rightarrow v = 10^{-6}c = 300$ m/s
l'observateur doit se rapprocher pour augmenter f

Canada 2005 Préparation Solution

If we forget about the internal energy sources of the Earth, the heat received from the Sun should equal the heat radiated from the surface. Denoting the constant in the Stefan-Boltzmann law with σ, surface temperatures of the Sun and the Earth with T_{Sun} and T_{Earth}, respectively, the surface areas with S_{Sun} and S_{Earth}, we may write : $Q_{radiated\ by\ sun} = \sigma T_{Sun}^4 S_{Sun}$, $Q_{radiated\ by\ Earth} = \sigma T_{Earth}^4 S_{Earth}$.

The surface areas are easy to calculate from the radii : $S_{Sun} = 4\pi R_{Sun}^2$, $S_{Earth} = 4\pi R_{Earth}^2$.

Luckily for us, not all the energy radiated by the Sun is absorbed by the Earth. We may assume that the fraction of absorbed energy roughly coincides with the fraction of the spherical surface Earth occupies at the radius of its orbit : $\frac{Q_{Earth\ accepts}}{Q_{radiated\ by\ Sun}} \approx \frac{\pi R_{Earth}^2}{4\pi D_{Earth\text{-}Sun}^2}$, where $D_{Earth\text{-}Sun}$ stands for the Earth's orbital radius.

With the energy balance formula, $Q_{Earth\ accepts} = Q_{radiated\ by\ Earth}$, and the numbers given we obtain : $\left(\frac{T_{Earth}}{T_{Sun}}\right)^4 \approx \frac{R_{Sun}^2}{4D_{Earth\text{-}Sun}^2}$, or $T_{Sun} \approx 4300\,K$. In fact, this is less than the actual Sun's temperature ($\approx 6000\,K$), which suggests that Earth reflects a considerable fraction of incoming radiation.

Canada 2005 Préparation problème 4 Solution

The diffraction (or interference) pattern in the problem consists of the dark and bright fringes on the photographic plate. Dark fringes correspond to the maximum intensities in this case.
To solve the problem we will use the equation for the distance Δx between two consecutive maximums on the screen for given distance l, slit separation d, and wavelength λ considering it the de Broglie wavelength of electron:

$$\Delta x = \frac{\lambda l}{d} \qquad (4.1)$$

According to the de Broglie theory $\lambda = \frac{h}{p}$ (4.2), where p is the momentum of electron.

The kinetic energy of electrons can be calculated as: $K = e\Delta V = 25$ eV. The energy is much less that the rest energy of electron (0.511 MeV), that is why the electrons in our problem are nonrelativistic. The relationship between momentum and kinetic energy is:

$$K = \frac{p^2}{2m} \qquad (4.3)$$

Combining eqs.4.1, 4.2, and 4.3 we can compose a formula to calculate Δx :

$$\Delta x = \frac{h}{p}\frac{l}{d} = \frac{h}{\sqrt{2mK}}\frac{l}{d} = \frac{h}{\sqrt{2me\Delta V}}\frac{l}{d} \qquad (4.4)$$

Calculation of eq.4.4 can be also simplified with the use of the value of the rest mass and the Plank's constant in electronvolts:

$$h = 4.14\text{x}10^{-15}\ \text{eV s}$$

$$\Delta x = \frac{hc}{\sqrt{2mc^2K}}\frac{l}{d} = \frac{4.14\times10^{-15}\times3\times10^{8}}{\sqrt{2\times0.511\times10^{6}\times25}}\frac{1}{50\times10^{-6}} = 4.91\times10^{-6}\,m = 4.91\,\mu m$$