Problem Set II

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1. (Iran, 1998)

$$\sqrt{x+y+z} \geq \sqrt{x-1} + \sqrt{y-1} + \sqrt{z-1} \quad \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2,\ x, y, z > 1\right)$$

2. (Hong Kong, 1998)

$$\sqrt{a-1} + \sqrt{b-1} + \sqrt{c-1} \leq \sqrt{c(ab+1)} \quad (a, b, c \geq 1)$$

3. (Romania, 1997)

$$\frac{x^9+y^9}{x^6+x^3y^3+y^6} + \frac{y^9+z^9}{y^6+y^3z^3+z^6} + \frac{z^9+x^9}{z^6+z^3x^3+x^6} \geq 2 \quad (xyz = 1,\ x, y, z > 0)$$

4. (Vietnam, 1991)

$$\frac{x^2y}{z} + \frac{y^2z}{x} + \frac{z^2x}{y} \geq x^2 + y^2 + z^2 \quad (x \geq y \geq z > 0)$$

5. (Ukrine, 1992)

$$\frac{a^2-b^2}{c} + \frac{b^2-c^2}{a} + \frac{c^2-a^2}{b} \geq 3a - 4b + c \quad (a \geq b \geq c \geq 0)$$

6. (Iran, 1997)

$$x_1^3 + x_2^3 + x_3^3 + x_4^3 \geq max\left(x_1 + x_2 + x_3 + x_4, \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4}\right)$$

$$(x_1x_2x_3x_4 = 1, x_1, x_2, x_3, x_4 > 0)$$

7. (Hong Kong, 2000)

$$\frac{1+ab^2}{c^3} + \frac{1+bc^2}{a^3} + \frac{1+ca^2}{b^3} \geq \frac{18}{a^3+b^3+c^3} \quad (abc = 1,\ a, b, c > 0)$$

8. (Poland, 1998)

$$abc + bcd + cde + def + efa + fab \leq \frac{1}{36}$$

$$\left(a+b+c+d+e+f = 1,\ ace + bdf \geq \frac{1}{108}\ a, b, c, d, e, f > 0\right)$$

9. (Italy, 1993)
$$a^2+b^2+c^2 \le a^2b+b^2c+c^2a+1 \ \ (0 \le a,b,c \le 1)$$

10. (Czech Republic, 2000)
$$(1-x^n)^m+(1-(1-x)^m)^n \ge 1 \ \ (m,n \in N,\ x \in [0,1])$$

11. (Hong Kong, 1997)
$$\frac{3+\sqrt{3}}{9} \ge \frac{xyz(x+y+z+\sqrt{x^2+y^2+z^2})}{(x^2+y^2+z^2)(xy+yz+zx)} \ \ (x,y,z>0)$$

12. (Poland, 1999)
$$a^2+b^2+c^2+2\sqrt{3abc} \le 1 \ \ (a+b+c=1,\ a,b,c>0)$$

13. (Czech-Slovak Match, 1999)
$$\frac{a}{b+2c}+\frac{b}{c+2a}+\frac{c}{a+2b} \ge 1 \ \ (a,b,c>0)$$

14. (Moldova, 1999)
$$\frac{ab}{c(c+a)}+\frac{bc}{a(a+b)}+\frac{ca}{b(b+c)} \ge \frac{a}{c+a}+\frac{b}{b+a}+\frac{c}{c+b} \ \ (a,b,c>0)$$

15. (Poland, 1993)
$$\frac{xy+xv+uy+uv}{x+y+u+v} \ge \frac{xy}{x+y}+\frac{uv}{u+v} \ \ (x,y,u,v>0)$$

16. (Ireland, 1997)
$$a^2+b^2+c^2 \ge abc \ \ (a+b+c \ge abc,\ a,b,c \ge 0)$$

17. (Austria-Poland, 1993)
$$\left(\frac{\sqrt{a}+\sqrt{b}}{2}\right)^2 \le \frac{a+\sqrt[3]{a^2b}+\sqrt[3]{ab^2}+b}{4} \le \frac{a+\sqrt{ab}+b}{3} \le \sqrt{\left(\frac{\sqrt[3]{a^2b}+\sqrt[3]{ab^2}}{2}\right)^3} \ \ (a,b \ge 0)$$

18. (Poland, 1991)
$$x+y+z \le 2+xyz \ \ (x^2+y^2+z^2=2,\ x,y,z \in R)$$

19. (Belarus, 1997)
$$\frac{a+y}{a+x}x+\frac{a+z}{a+x}y+\frac{a+x}{a+y}z \ge x+y+z \ge \frac{a+z}{a+z}x+\frac{a+x}{a+y}y+\frac{a+y}{a+z}z \ \ (a,x,y,z>0)$$

20. (Turkey, 1999)

$$(a+3b)(b+4c)(c+2a) \geq 60abc \ \ (c \geq b \geq a \geq 0)$$

21. (APMO, 1998)

$$\left(1+\frac{a}{b}\right)\left(1+\frac{b}{c}\right)\left(1+\frac{c}{a}\right) \geq 2\left(1+\frac{a+b+c}{\sqrt[3]{abc}}\right) \ \ (a,b,c>0)$$

22. (Lithuania, 1987)

$$\frac{x^3}{x^2+xy+y^2}+\frac{y^3}{y^2+yz+z^2}+\frac{z^3}{z^2+zx+x^2} \geq \frac{x+y+z}{3} \ \ (x,y,z>0)$$

23. (Russia, 1995)

$$\frac{1}{xy} \geq \frac{x}{x^4+y^2}+\frac{y}{y^4+x^2} \ \ (x,y>0)$$

24. (Latvia, 1997)

$$\frac{1}{a+b}+\frac{1}{a+2b}+\cdots+\frac{1}{a+nb} < \frac{n}{\sqrt{a(a+nb)}} \ \ (n \ inN, \ a,b,c>0)$$

25. (Austria-Poland, 1995)

$$(n-1)(m-1)(x^{n+m}+y^{n+m})+(n+m-1)(x^n y^m+x^m y^n) \geq nm(x^{n+m-1}y+xy^{n+m-1})$$

$$(m,n \in N, \ x,y>0)$$

26. (Klamkin's Inequality)

$$\frac{1}{(1-x)(1-y)(1-z)}+\frac{1}{(1+x)(1+y)(1+z)} \geq 2 \ \ (-1<x,y,z<1)$$

27. (Carlson's Inequality)

$$\sqrt[3]{\frac{(a+b)(b+c)(c+a)}{8}} \geq \sqrt{\frac{ab+bc+ca}{3}} \ \ (a,b,c>0)$$

28. (Nesbitt's Inequality)

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b} \geq \frac{3}{2} \ \ (a,b,c>0)$$

29.

$$\frac{x}{1+x^2}+\frac{y}{1+y^2}+\frac{z}{1+z^2} \geq \frac{2x(1-x^2)}{(1+x^2)^2}+\frac{2y(1-y^2)}{(1+y^2)^2}+\frac{2z(1-z^2)}{(1+z^2)^2} \ \ (xy+yz+zx=1, \ x,y,z>0)$$

30.

$$x^y + y^x > 1 \ \ (0 < x, y < 1)$$

31.

$$\frac{a}{bc} + \frac{b}{ca} + \frac{c}{ab} \geq \frac{2}{a} + \frac{2}{b} - \frac{2}{c} \ \ (a, b, c > 0)$$

32.

$$\sqrt{\frac{a^2 + b^2 + c^2 + d^2}{4}} \geq \sqrt[3]{\frac{abc + bcd + cda + dab}{4}} \ \ (a, b, c, d > 0)$$

33.

$$a^4 + b^4 + c^4 + d^4 + e^4 \geq a + b + c + d + e \ \ (abcde = 1, \ a, b, c, d, e > 0)$$

34.

$$\frac{a^3 + b^3 + c^3}{a + b + c} + \frac{b^3 + c^3 + d^3}{b + c + d} + \frac{c^3 + d^3 + a^3}{c + d + a} + \frac{d^3 + a^3 + b^3}{d + a + b} \geq a^2 + b^2 + c^2 + d^2 \ \ (a, b, c, d > 0)$$

35.

$$|x| + |y| + |z| + |x + y + z| \geq |x + y| + |y + z| + |z + x| \ \ (x, y, z \in C)$$

36.

$$a^3 + b^3 + c^3 + 3abc \geq ab(a + b) + bc(b + c) + ac(a + c) \ \ (a, b, c > 0)$$